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Shielded Composite Dielectric Spherical Resonator

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Abstract

Electromagnetics theoretic analysis of composite Shielded dielectric resonator has been made. Characteristic equation for the TE and TM modes has been derived shielded composite dielectric sphere of radius of the order of μm size are found suitable for the optical frequency reason where as for the microwave reason. Radius of the order of mm size are founds suitable for the optical frequency region where as the microwave region.Expressions for the quality factor for realistic resonator i.e. for a composite shielded dielectric resonator with non-zero conductivity and a metal shield with a finite conductivity have also been derived for TE and TM modes. Computations for the quality factors have been made for resonators with parameter suitable for the optical and micro wave reason.

Keywords: Spherical Resonator, Quality Factors, Eigen Modes.

Introduction

The electromagnetic field analysis of the shielded spherical dielectric resonators starting from the Maxwell's equationsthe problem has been treated using spherical coordinate's geometry. Assuming and idealized situation in which the shield is perfectly conducting (Loss less) and the dielectric is (loss less) the problem has been treated separately for the TE and TM modes. The field for the expression has been derived and using the boundary conditions.Characteristics equations have been derived. Since the characteristic equations are transcendental equation these have been solved numerically to calculate resonant frequency. In fact the shields as well as dielectric are lossed due to their finite conductivities. Hence, the quality factor for a realistic resonator is finite. Expressions for the quality factor have been derived by using expressions for the dielectric loss. In order to calculate quality factor the resonant frequency for the ideal resonator have been used. Effect of variation of the sphere radius on the resonant frequency and quality factor has been study in this paper.

Theory

A Shielded composite dielectric spherical resonator consist of two concentric spheres i.e. solid inner dielectric sphere with radius a and permittivity ϵ_1 and outer dielectric spherical shall with radius b and permittivity ϵ_2. The outer dielectric spherical shall is shielded by a perfectly conducting metal case of radius b. The dielectrics of the two spheres are non-magnetic i.e.$\mu_1 = \mu_0 = \mu_2$ electrically homogeneous and isotropic. Thus the resonator consists of two dielectric regions.

One in the region $0 \le r \le a$ and the other in the region $a \le r \le b$. The solution of the radial part of the wave equation in the two region can be written as

$$X(r) = AJ_{n+1/2}(k_1 r) \quad \text{for } 0 \le r \le a \qquad (1)$$

$$X(r) = BJ_{+1/n2}(k_2 r) + CY_{n+1/2}(k_2 r) \quad \text{for } a \le r \le b \qquad (2)$$

Where

$K1 = \omega/c \sqrt{\epsilon^r_1} = k_0 \sqrt{\epsilon^r_1}$ and $K2 = \omega/c \sqrt{\epsilon^r_2} = k_0 \sqrt{\epsilon^r_2}$ and A, B and C are Constants, and $\epsilon^r_2 = \epsilon_2/\epsilon_0$, ω is angular frequency and c is speed of light in vacuum. The symbols $J_v(Kr)$ adn$Y_v(kr)$ $(\nu = n + \frac{1}{2}$ and $k = k_1/k_2)$ appearing in the equations. (1) and (2) are respectively the Bessel functions of the first and second kinds. V brings the order of the function and the symbols A, B and C are constants. Apart from a normalization constant the complete solution of the wave equation is given by,

$$\Psi(r, \theta, \varphi) = A/\sqrt{k_{1r}} J_{n+1/2}(k_{1r}) P_n^m(\cos\theta) \cos m\varphi,$$

for $0<r<a$ (3)

$\Psi(r\theta,\varphi)=(B/\sqrt{k_{2r}}J_{n+1/2}(k_2,r+ \quad C/\sqrt{k_2 r} \quad Y_{n+1/2}(k_2,r))P_n^m$ $(\cos\theta)\cos m\varphi$, for a $\leq r\leq$ b (4)

where ψ(*r*, θ, φ) is the amplitude of the electromagnetic radiation (electric/magnetic) field, *r*, θ, $_\Delta$ are the co-ordinates in the spherical polar co-ordinates – system, P_n^m (cosθ) is the associated Legendre polynomial of order m and degree n and J and Y are the Bessel's functions (of the order n + $\frac{1}{2}$) of the first and second kinds respectively. Using the expressions for ψ(*r*, θφ,) from the equation (3) and (4) are can find out the expressions for the electric and magnetic field components for the TE_{nml} and TM_{nml} modes separately.

Characteristic equation for theTE_{nml} andTM_{nml}

The expressions for the electric and magnetic field's foe the TE_{nml} and TM_{nml} in the region 0 ≤*r*≤*a* and 0 ≤*r*≤ b are known. One can derive the characteristic equations for the TE_{nml} and the TM_{nml} modes, employing the boundary conditions on the dielectric surface (*r* = *a*) and the boundary conditions on the metallic surface (*r* = b). The tangential components of E and H are continuous at the surface (r = a) separating the two electric media and the tangential components of the electric field vector is zero at the metallic surface (r = b) using the expressions for the fields components and Apply the Bessel's the characteristic equations for the TE_{nml} modes.

$K_1 j_{n-1}$ (k_1 a) {$j_n(k_2$ a) y_n (k_2 b) – Jn (k2 b) y_n (k_2 a)} + $k_2 J_n$ (k_1 a) {y_{n-1} (k_2 a) j_n (k_2 b) – Yn (k_2b) J_{n-1} (k_2 a) =0 (5)

Similar to the case of the TEnml modes, one can find the characteristics equations for TMnml modes using the boundary condition and the field expression in the two regions at r = a and r = b the characteristic equation for TMnml modes one get

K^2_2 ab $\in r_1 j_n$ (k_1a)) {$j_{n-1}(k_2$a) y_{n-1} (k_2 b) – j_{n-1} (k_2 b) y_{n-1} (k_2 a)} + $k_1k_2$ab$\in_{r2}$ j_{n-1} (k_1 a) {y_n (k_2 a) j_{n-1} (k_2 b) –y_{n-1} (k_2 b) j_n (k_2 a)} + n k_2 a $\in_{r1} j_n$ (k_1 a) {y_{n-1} (k_2 a) j_n (k_2 b) – y_n (k_2 b) j_{n-1} (k_2 a)} + n k_1 a $\in_{r2}$ j_{n-1} (k_1 a) {j_n (k_2 a) y_{n-1} (k_2 b) – y_{n-1} (k_2 b) j_n (k_2 a)} + n k_2 b $\in_{r1} j_n$ (k_1 a) {y_n (k_2 a) j_{n-1} (k_2 b) – y_{n-1} (k_2 b) j_n (k_2 a)} + n k_2 b $\in_{r2} j_n$ (k_1 a) {j_n (k_2 a) y_{n-1} (k_2 b) – j_{n-1} (k_2 b) y_n (k_2 a)} + $n^2 \in_{r1} j_n$ (k_1 a) {j_n (k_2 a) y_n (k_2 b) – j_n (k_2 b) y_n (k_2 a)} + $n^2 \in_{r2} j_n$ (k_1 a) {y_n (k_2 a) j_n (k_2 b) – y_{n-1} (k_2 b) j_n (k_2 a)}= 0 (6)

Energy Losses and Quality Factor

The expression for the energy W stored in the shielded dielectric sphere given by

$$W = \frac{1}{2} \in \iiint_v E.E^* \, dv = \underline{1/}2 \iiint_v H.H^* \, dv$$

(7)

If the sum of the energy W_1 stored inside the inner dielectric sphere (0≤ r ≤ a) and the energy W_2 stored within the outer dielectric spherical shall (a≤ r ≤ a) for TE_{mml} and TM_{nml} respectively.

Due to the loss natures of the dielectrics and metallic shield there is energy loss associated with the composite shielded resonator.

Computations of the Resonant Frequencies and Quality Factors Results And Discussions

In this case also o a mode with a given value of n is n + 1 fold degenerate i.e. (n + 1) modes have some frequencies e.g. TE_{10l} and TM_{11l} have some resonant frequencies for a given value of l. Where l is the order of the root of the characteristic equation. We have determine five roots (l = 1 – 5) for each of the characteristic equations on 5 and 6 for dielectric materials with $\in_1^r$= 4.0 and ($\in_2^r$= 3.78 (for coring) glass $\in_1^r$= 4.0 and (for quartz) $\in_2^r$= 3.78) and 0.1 – 0.9 $_\mu$m and b=1.0 – 10.0 $_\mu$m. Table 1 (a - c) and Table 2 (a - c) so variation of the resonant frequency with the radius a of the inner dielectric sphere for TE_{nml} and TM_{nml} (n = 1 – 3, l = 1 - 5) modes respectively. However, variation of the radius a of the inner dielectric sphere does not have noticeable effect on the resonant frequency and quality factor. This is because the permittivity of the inner and outer dielectric spheres are very close in the present case. ($\in_1^r$ =4.0 for corning glass and $\in_2^r$= 3.78 for quartz) As there are no dielectric materials available which are transparent in the optical range having high value of the permittivity. Composite dielectric spherical resonator are not of much use in optical region. In such a case if $\in_1^r$ and $\in_2^r$ are widely differing charging radius (a) of the inner sphere will affect the resonant frequency and quality factor. Table3(a-b) so the variation of the quality factorincreases with the increasing b and has magnitude in the range 10-10 contrary to the case of optical region where variation of a has little effect on the quality factor and resonant frequency .Owing to the closer magnitudes of the permittivity of the two dielectrics, in this case however, the quality factor depends on the inner radius(a) strongly due to widely differing permittivity's of the two spheres.

Table 1 (A) Variation of Resonant Frequency (υ_1^+ml) with a for TE_{10l} Modes

a(*mm*) l	1	2	3	4	5
0.1	1.102	1.895	2.678	3.451	4.224
0.3	1.102	1.885	2.654	3.422	4.195
0.5	1.093	1.866	2.644	3.403	4.171
0.7	1.079	1.861	2.625	3.384	4.137
0.9	1.069	1.847	2.606	3.370	4.119

+ Frequencies are given by the numbers under the columns multiplied by 10^{14} Hz, the value of b = 1.0 µm, $\in_1^r$= 4.0 and $\in_2^r = 3.78$

Table 1(B) Variation of Resonant Frequency (υ_2^+ ml) with A for TE_{20l} Modes

a(μ*m*) l	1	2	3	4	5
0.1	1.417	2.229	3.026	3.809	4.567
0.3	1.417	2.229	3.007	3.775	4.553
0.5	1.408	2.200	2.988	3.756	4.525
0.7	1.389	2.194	2.964	3.737	4.493
0.9	1.375	2.172	2.945	3.718	4.467

+ Frequencies are given by the numbers under the columns multiplied by 10^{14} Hz, the value of B = 1.0 μm, $\in_1^r$= 4.0 and $\in_2^r = 3.78$

Table 1(C) Variation of Resonant Frequency (υ_3^+ ml) with a for TE_{30l} Modes

+ Frequencies are given by the numbers under the columns multiplied by 10^{14} Hz, the value of b = 1.0 μm, $\in_1^r$= 4.0 and $\in_2^r = 3.78$

Table 2(A) Variation of Resonant Frequency (υ1+ ml) with a for TM10l Modes

a(a*m*) l	1	2	3	4	5
0.1	0.663	1.499	2.281	3.055	3.828
0.3	0.663	1.480	2.267	3.049	3.785
0.5	0.644	1.470	2.238	3.037	3.774
0.7	0.630	1.437	2.197	3.010	3.742
0.9	0.621	1.394	2.181	2.963	3.710

+ Frequencies are given by the numbers under the columns multiplied by 10^{14} Hz, the value of b = 1.0 μm, $\in_1^r$= 4.0 and $\in_2^r = 3.78$

Table 2(B) Variation of Resonant Frequency (υ_2^+ ml) with a for TM_{20l} Modes

a(υ*m*) l	1	2	3	4	5
0.1	1.217	2.138	2.959	3.756	4.544
0.3	1.217	2.129	2.930	3.751	4.521
0.5	1.203	2.099	2.667	3.638	4.492
0.7	1.160	2.075	2.640	3.610	4.452
0.9	1.127	2.035	2.613	3.561	4.917

+ Frequencies are given by the numbers under the columns multiplied by 10^{14} Hz, the value of b= 1.0 μm, $\in_1^r$= 4.0 and $\in_2^r = 3.78$

Table 2 (C) Variation of Resonant frequency (υ_3^+ ml) with a for TM_{80l} modes

a(υ*m*) l	1	2	3	4	5
0.1	0.950	1.823	2.625	3.413	4.191
0.3	0.945	1.804	2.611	3.392	4.187
0.5	0.926	1.789	2.600	3.360	4.161
0.7	0.888	1.759	2.523	3.330	4.127
0.9	0.876	1.729	2.501	3.301	4.087

+ Frequencies are given by the numbers under the columns multiplied by 10^{14} Hg, the value of B = 1.0 μm, $\in_1^r$= 4.0 and $\in_2^r = 3.78$

Table 3 (A) Variation of Q-Factor with b for TE_{10l} Modes

b(m*m*) l	1	2	3	4	5
2.0	1961.0	1370.0	859.17	795.86	570.81
4.0	4004.0	1444.0	1290.0	908.0	667.10
6.0	5199.0	1894.0	1389.0	1207.0	929.0
8.0	9064.0	2393.0	1615.0	1492.0	1185.0
10.0	12131.0	3012.0	1947.0	1563.0	1402.0

A=0.72 Mm, $\in_1^r$=36 And $\in_2^r = 3.78$

Table 3 (B) Variation of Q-Factor with b for TM_{10l} Models

b(m*m*) l	1	2	3	4	5
2.0	1109.17	577.66	390.66	317.64	268.59
4.0	1173.91	709.39	543.46	413.02	375.96
6.0	1190.35	962.09	633.26	553.79	444.80
8.0	1393.55	1136.98	841.63	606.82	562.01
10.0	1739.89	1178.69	1021.93	754.58	595.83

a= 0.72 mm,$\in_1^r$=36 and $\in_2^r = 3.78$

Conclusion

Electromagnetic field analysis of spherical dielectric resonator has been presented by a number of workers. In all the cases the resonant frequencies and quality factors have been computed for the resonators with parameters suitable for the microwave region. From the survey of the published literature it. Seems that no theoretical and or experimental studies are available for the optical frequency region. In addition neither the normal mode frequency nor the quality factors in the optical region seem to have been reported for the shielded spherical dielectric resonators. Shielded resonators have drastically reduced quality factors due to metallic loss of the shield and dielectric loss of that dielectric medium. It discusses the resonant frequency advantages and various limitations of communication system. Finally it shows all the future aspects which will be in the market. Out of which same became absolute and same are still in use for research and for developing general application.

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